

# An Efficient Tensor Based Decomposition of Hyperspectral Image Representation

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**Abstract** - In this paper, we consider decomposition techniques for hyperspectral images using tensor based methods. Hyperspectral images often contain reflectance information from numerous wavelengths. Thus, efficient decomposition methods are required to compress the data and make it further suitable for other processes such as classification and pattern recognition. We present a simple and efficient multilinear principal component analysis method for hyperspectral image decomposition and compare the performance between the proposed tensor (MPCA) and matrix (PCA) based decomposition methods. Simulation results shows that performance does not degrade when tensor based methods are used for decomposition. Extensive simulation results show that tensor based decomposition of hyperspectral image storage requires reduced memory at reduced computation time.

**Keywords**—Hyperspectral, Tensor, Decomposition, MPCA (Multilinear Principal Component Analysis)

## I. INTRODUCTION

Hyperspectral images make use of the different characteristics such as molecular composition and shape due to which each material absorb, reflect and emit electromagnetic radiation uniquely [1]. Hence, if the radiation is calculated at different wavelengths for different objects, it can be used to effectively distinguish the various objects in a scene without coming into physical contact with it. Hyperspectral images have been used extensively over various domains such as pharmaceuticals, agriculture, food, remote sensing, biotechnology, security and defense etc. Thus, hyperspectral imaging provides a space of interest for further research and exploration [2][3].

Hyperspectral images such as those captured by Stanford Center for Image systems Engineering (SCIEN) make use of imaging spectrometers to collect and present an image taken across various frequencies giving it a huge size when represented as a tensor [4]. The accuracy of classifying hyperspectral images reduces drastically when they are used without decomposition of the original data. Popular decomposition techniques such as Principal Component Analysis (PCA), Discrete Wavelet Transform (DWT) are used to operate on two dimensional images. Thus, if one of these

techniques were applied to a three dimensional hyperspectral data, vectorization of data is to be performed before applying the reduction. This does not take into account the spectral arrangement of the images across the hyperspectral data [5].

A tensor object in machine vision applications is usually specified in high dimensional tensor space. Recognition or classification methods operating on such high dimensional tensor space suffer from what is known as, Curse of Dimensionality. Handling high dimensional samples are computationally expensive and results in reduced classification accuracy. It also reduces computational complexity and time for further processes with the data in regression, classification etc. Thus, dimensionality reduction aims to transform high dimension data to lower dimensions while maintaining the vital information present in the data [6].

Principal Component Analysis (PCA) is commonly used for dimensionality reduction by performing orthogonal transformation on a set of correlated variables to transform into a set of uncorrelated linear variables. However, vectorizing a tensor of size such as (400x200x20) will result in a matrix of size (160000x1) which will be computationally arduous. To overcome this difficulty, it is necessary to take advantage of the natural tensor structure of the data by applying Multilinear Principal Component Analysis (MPCA) for data compression. MPCA acts similar to Higher Order Singular Value Decomposition (HOSVD) by seeking low dimensional multilinear projections of tensor objects that capture the maximal data variation by exploiting the natural structure of the tensor.

The remaining sections of this paper are organized as follows. Section II briefly introduces the principal component analysis for image decomposition. Section III provides the multilinear principal component analysis algorithm. Section IV presents experimental procedure, Section V gives the experimental result and Section VI summarizes this paper.

## II. PRINCIPAL COMPONENT ANALYSIS (PCA)

Principal Component Analysis transforms correlated data into linear uncorrelated data of reduced dimensions while

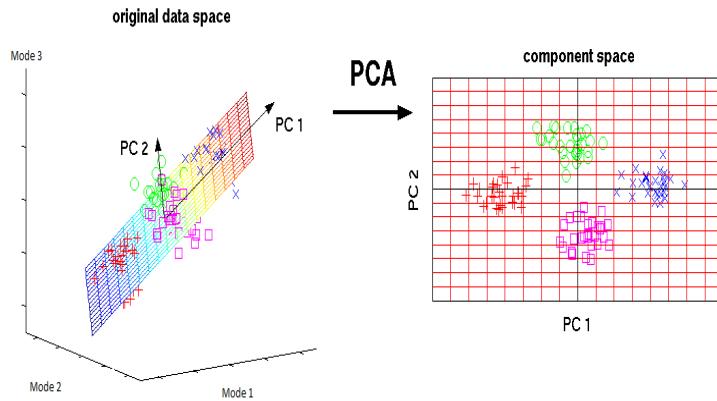


Figure 1. PCA transformation.<sup>[7]</sup>

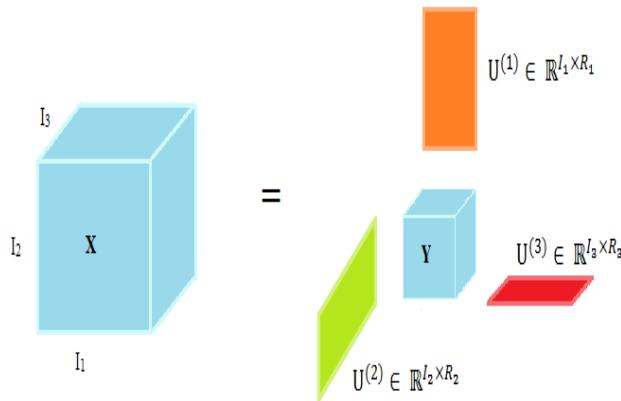


Figure 2. Tensor decomposition

retaining the original structure of the data and the pixels or samples which contain the highest variance [8].

Consider a random vector population  $x$ , where

$$x = (x_1, \dots, x_n)^T \quad (1)$$

where the mean and covariance are  $\mu_x = E\{x\}$  and  $C_x = E\{(x - \mu_x)(x - \mu_x)^T\}$  respectively. Now the eigen vectors ( $e_i$ ) and eigen values ( $\lambda_i$ ) are found for the covariance matrix

$$C_x e_i = \lambda_i e_i, i = 1, \dots, n \quad (2)$$

Thus, the solutions are found for the characteristic equation  $|C_x - \lambda I| = 0$ . Let  $H$  be a matrix containing the eigen vectors of the covariance matrix as the row vectors,

Transform the data vector  $x$  to get,

$$y = H(x - \mu_x) \quad (3)$$

The original data vector can be reconstructed as,

$$x = H^T y + \mu_x \quad (4)$$

Now, choose the first 'K' eigen vectors to be considered which transforms the equations as

$$y = H_K(x - \mu_x) \quad (5)$$

$$x = H_K^T y + \mu_x \quad (6)$$

Thus, the image is transformed into the new subspace.

### III. MULTILINEAR PRINCIPAL COMPONENT ANALYSIS (MPCA)

MPCA is a tensorial, i.e., multilinear extension of PCA and it acquires original tensorial input variations better than PCA. In this section, MPCA algorithm used to solve the problem of dimensionality reduction for tensor objects is given in brief.[9]

#### ALGORITHM:

**INPUT:** A tensor of N-dimensions

$$\{x_m \in \mathbb{R}^{I_1 \times I_2 \times \dots \times I_N}, m = 1, \dots, M\} \quad (7)$$

**OUTPUT:** Low dimensional representation of the input tensor i.e output

$$\{Y_m \in \mathbb{R}^{P_1 \times P_2 \times \dots \times P_N}, m = 1, \dots, M\} \quad (8)$$

**Step1:** Center the input samples as  
Where

$$\tilde{\chi} = \frac{1}{M} \sum_{m=1}^M \chi_m \quad (9)$$

**Step2:** Calculate the eigen decomposition of mode-n total scatter matrix in full projection using

$$\Phi^{(n)*} = \sum_{m=1}^M \tilde{\chi}_{m(n)} \cdot \tilde{\chi}_{m(n)}^T \quad (10)$$

**Step3:** Set the projection matrix  $\tilde{U}^{(n)}$  to consist of eigen vectors corresponding to the most significant  $P_n$  eigen values, where  $n=1,2,\dots,N$

**Step4:** calculate

$$\{\tilde{Y}_m = \tilde{\chi}_m \times_1 \tilde{U}^{(1)\top} \times_2 \tilde{U}^{(2)\top} \dots \times_N \tilde{U}^{(n)\top}, m = 1, \dots, M\} \quad (11)$$

$$\Psi_{y_0} = \sum_{m=1}^M \|\tilde{Y}_m\|_F^2$$

**Step5:** For local optimization, iterative process is carried as follows

For  $k=1,2,\dots,K$

For  $n=1,2,\dots,N$

-Set the projection matrices  $\tilde{U}^{(n)}$  to consist of eigen vectors corresponding to the largest eigen values.  
where,  $n=1,2,\dots,N$

Calculate  $\{\tilde{Y}_m, m = 1, \dots, M\}$  and  $\Psi_{y_k}$

If,

$$\Psi_{y_k} - \Psi_{y_{k-1}} < \eta \quad (12)$$

break

else repeat step5.

**Step6:** The feature tensor after projection is obtained as

$$[Y_m = \chi_m \times_1 \tilde{U}^{(1)\top} \times_2 \tilde{U}^{(2)\top} \dots \times_N \tilde{U}^{(N)\top}, m = 1, \dots, M] \quad (13)$$

,where

#### IV. EXPERIMENT

The image used in this experiment has been got from the SCIEN (Stanford Center for Image Systems Engineering) depicting a view of California across 148 different wavelengths, ranging from 400nm to 950nm. The original input image is stored in a tensor of size (702x1000x148) where 702, 1000 represent the number of rows and columns of pixels respectively. 148 represents the number of spectral samples taken. The simulations make use of Tensor Toolbox [10][11].

The hyperspectral images, in tensor format, are decomposed using two methods namely, PCA and MPCa. For PCA, the following steps are taken. To begin with, the Hyperspectral tensor is loaded and converted into a matrix by n-mode flattening [12]. The PCA algorithm is implemented by defining the number of principal components to be considered. Then, the original tensor is reconstructed from the coefficient matrix by feeding the predefined number of principal components. The SNR of the output tensor is measured and the error is calculated on a pixel to pixel basis. The above steps are repeated using a different number of principal components.

For MPCa, the following steps are taken. The input hyperspectral image is loaded into a tensor of size (702x1000x148). The function MPCa is called for with the input tensor, ground truth, variation and number of iterations as input. From the resulting factor matrices for the 1<sup>st</sup> and 2<sup>nd</sup> mode and core tensor, the original tensor is reconstructed.

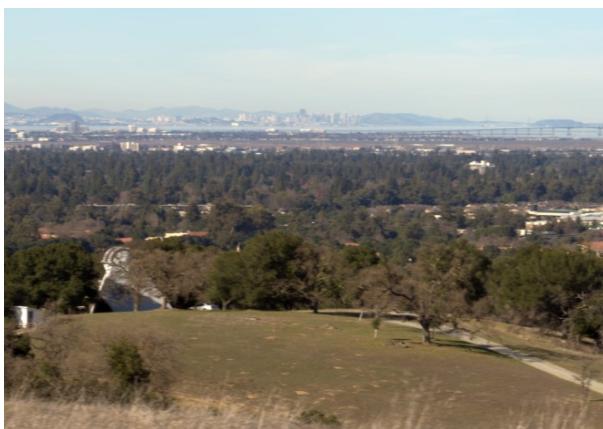


Figure 3. RGB rendition of input image <sup>[4]</sup>



Figure 4. Colorbar for Hyperspectral image

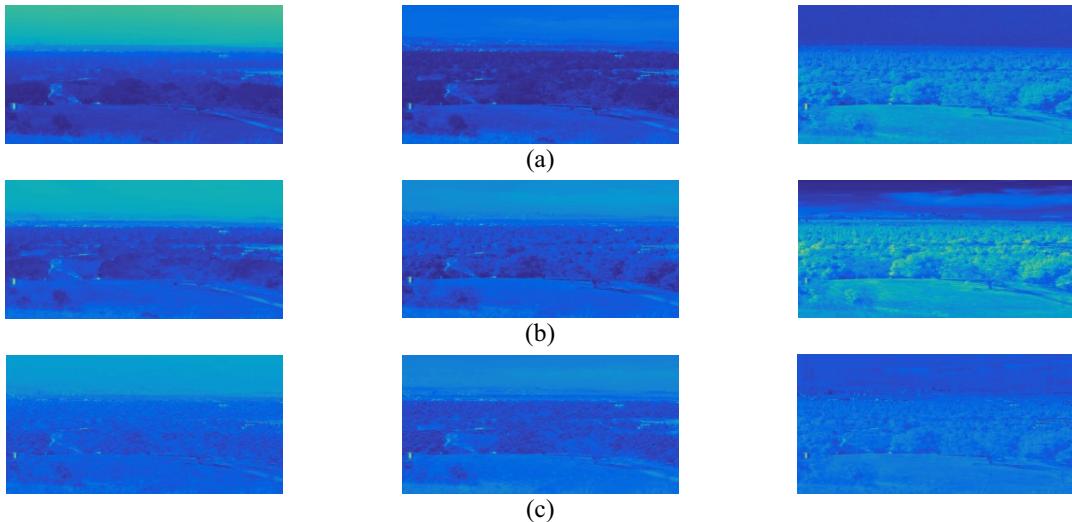


Figure 5 Row (a) contains colour scaled versions of the original hyperspectral image at 3 wavelengths, namely; 414.7243nm, 684.4339 nm and 950.4988 nm. Row (b) contains the corresponding colour scaled versions of the reconstructed hyperspectral image obtained after applying PCA. Row (c) contains the colour scaled versions of the reconstructed hyperspectral image obtained after applying MPCa.

TABLE I. SNR, size of feature tensor and Relative error for different amounts of variation kept in each mode while applying MPCa

Amount of variation kept in each mode	95%	96%	97%	98%	99%
SNR (dB)	20.2031	21.0289	22.2045	23.7988	26.5725
Size of feature tensor	13x10x148	18x15x148	27x23x148	42x37x148	79x72x148
Relative error (%)	15.1750	13.7949	11.9983	9.9587	7.3370

TABLE II. Computational time for MPCa and PCA with similar SNR performed using Intel i7-4500U processor

Algorithm	Computational time	
	SNR=22dB	SNR=27dB
MPCA	13.936887s	15.390155s
PCA	14.257096s	15.657825s

The SNR of the output tensor is measured and the error is calculated on a pixel-to-pixel basis. The above procedure is repeated by changing the variation value in the function.

Colour scaled versions of the original image is shown in Figure 5(a). The colour scaled versions of the reconstructed image after decomposition by PCA and the MPCa techniques are illustrated in Figures 5(b) and 5(c). It is evident that no

significant distortion is observed on reconstructing the image after subjecting it to MPCa based decomposition. The relative error at different values of SNR have been tabulated in Table I. The computational time for MPCa to achieve the same SNR as PCA is tabulated in Table II. It can be observed that MPCa is quicker than the PCA technique.

## VI. CONCLUSION

In this paper, hyperspectral images are decomposed using two different methods. PCA which is a two-dimensional decomposition method neglects spatial rearrangement. Thus, Multilinear Principal Component Analysis (MPCA) is applied which works on the data as a tensor, maintaining spatial rearrangement. The data also shows that MPCA is marginally computationally less intensive than PCA. The simulation results show that the change in variance has a great effect on SNR and relative error in an image. The MPCA, backed by verification results thus provides an excellent way to decompose hyperspectral images.

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